NUMERICAL ANALYSIS
Assignment -5 (week 5)
Total Marks - 25
Posted on - 21/8/2017 (Monday);
To be submitted on or before-30/8/2017 (Wednesday), 23.59 hours.

Problems on

- Gauss quadrature three point method.
- Numerical Solution of ODE:
  Taylor series Methods,
  Euler's Method.

INSTRUCTIONS

- This is a question paper cum answer booklet.
- Take a print out of this.
- Present the details of the computations of the solution of each problem **which you will have to show** in the space provided at the bottom of the page.
- Fill in the answers in the space provided below each question.
- Scan the booklet and submit it as a pdf file before the deadline for evaluation.

1. Use Gaussian three-point quadrature formula to evaluate $\int_0^4 \frac{sint}{t} dt$ and then Fill in the blanks.

   (a) If $\int_0^4 \frac{sint}{t} dt = \int_{-1}^1 F(x)dx$, then$F(x) =$ _____,
   (b) $\int_0^4 \frac{sint}{t} dt \simeq$ _____ . (2+2=4 marks)

Show your work for the solution of problem 1 in the space provided below.

2. Solve $y' = xy + y^2 - 2$; $y(0) = 1$ using Euler's method with step-size $h = 0.5$ and estimate $y(1)$.
Fill in the blanks:
(a) $y(0.5) \simeq$ _____,

(b) $y(1) \simeq$ _____ . (2+2=4 marks)

Show your work for the solution of problem 2 in the space provided below.

3. Use Taylor-series method of order 2 to solve $y' = xy + y^2 - 2$, $y(0) = 1$ with step-size $h = 0.5$ and estimate $y(1)$.
Fill in the blanks:
(a) $y'(0) =$ _____, (b) $y''(0) =$ _____,

(c) $y(0.5) \simeq$ _____, (d) $y'(0.5) =$ _____

(e) $y''(0.5) =$ _____, (f) $y(1.0) \simeq$ _____ . (6 × 1=6 marks)

Show your work for the solution of problem 3 in the space provided below.

4. (a) Using Taylor-series method of order 3, determine $y(1.3)$ given that $\frac{dy}{dx} = xy^{\frac{1}{3}}$; $y(1) = 1$, taking step-size $h = 0.3$.
(b) Solve $\frac{dy}{dx} = xy^{\frac{1}{3}}$; $y(1.2) = 1.22787$ and determine $y(1.3)$ using Taylor series method of order 3.
Fill in the blanks:
(a)$y'(1) =$ _____, $y''(1) =$ _____,

$y'''(1) =$ _____, $y(1.3) =$ _____ . (1+2=3 marks)

(b)$y'(1.2) =$ _____, $y''(1.2) =$ _____,

$y'''(1.2) =$ _____, $y(1.3) =$ _____ . (1+2=3 marks)

Show your work for the solution of problem 4 in the space provided below.

5. Consider the IVP $y' = y + x; y(1) = 2$. If the error in $y(x)$ obtained from the first four terms of the Taylor series is to be less than 5×10^{-4} after rounding, find x.
Fill in the blanks:
(a)$y'(1) =$ _____, (b) $y''(1) =$ _____,

(c) $y'''(1) =$ _____, (d) $R_4 =$ _____,

(e) $x \simeq$ _____ . (5×1=5 marks)

Show your work for the solution of problem 5 in the space provided below.